Math 60 10.5 Graphing Quadratic Functions Using Properties
A.K.A. Vertex Formula, Intercepts, Max or Min

(2 days)

Objectives

day 1
1) Find the vertex of a quadratic function using the vertex formula
2) Find the x-intercepts of a quadratic function
3) Use the discriminant to determine the number of x-intercepts of a quadratic function.
4) Find the y-intercepts of a quadratic function.
5) Graph quadratic functions in the form $f(x) = ax^2 + bx + c$

day 2.
6) Find the minimum value of an upward-opening parabola (quadratic function)
7) Find the maximum value of a downward-opening parabola (quadratic function).
8) Use max or min to solve applications.

Derivation of Vertex formula (skip in class)

We used completing the square to write $f(x) = ax^2 + bx + c$ in the form $f(x) = a(x-h)^2 + k$, and this can be done using general algebra to get a formula for h (and k).

$f(x) = ax^2 + bx + c$

$f(x) = a(x^2 + \frac{b}{a}x \qquad) + c$ factor out a

complete the square
$\# = \frac{b}{a} \div 2 = \frac{b}{a} \cdot \frac{1}{2} = \frac{b}{2a}$ ← use $\frac{b}{2a}$ for factor

$\#^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$ ← add $\frac{b^2}{4a^2}$ to inside ( ).

$f(x) = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a \cdot \frac{b^2}{4a^2}$ ← subtract $\frac{b^2}{4a^2} \cdot a$
(distribute a)

$f(x) = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$

x coord of vertex $-\frac{b}{2a}$

y-coord of vertex $c - \frac{b^2}{4a}$
or just $f\left(-\frac{b}{2a}\right)$

Math 60 10.5 – 1st

When we use the word "quadratic", we could mean

<u>Quadratic Equation</u> $ax^2 + bx + c = 0$

one variable only.
2 solutions (max) $x = \#$, $x = \#$

or <u>Quadratic Function</u> $f(x) = ax^2 + bx + c$

or $y = ax^2 + bx + c$

2 variables (x, y)
values that satisfy the function form ordered pairs, which can be graphed to make a parabola.

The quadratic <u>equation</u> is related to the quadratic <u>function</u>
because if we set $y = 0$ (or $f(x) = 0$)
in the quadratic function

we get

- the x-intercepts of the graph of the quadratic function (where parabola crosses x-axis)
- the quadratic equation

But the two are different and should not be confused with each other.

Math 60 10.5

> Memorize: $f(x) = ax^2 + bx + c$
> has vertex x-coord $h = \frac{-b}{2a}$
> and vertex y-coord $k = f\left(\frac{-b}{2a}\right)$ ← Just plug in x-result.

Find the vertex, and rewrite function in the form $f(x) = a(x-h)^2 + k$

① $f(x) = x^2 - 2x + 3$

$h = \text{x-coord} = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

$k = \text{y-coord} = f(1) = 1^2 - 2(1) + 3 = 2$

$\boxed{\text{vertex } (1, 2)}$ $\boxed{f(x) = (x-1)^2 + 2}$ a = 1 unchanged

② $g(x) = x^2 - 2x - 3$

$h = \text{x-coord} = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

$k = \text{y-coord} = f(1) = 1^2 - 2(1) - 3 = -4$

$\boxed{\text{vertex } (1, -4)}$ $\boxed{f(x) = (x-1)^2 - 4}$ a = 1 unchanged

③ $h(x) = 2x^2 - 6x + 7$

$h = \text{x-coord} = \frac{-b}{2a} = \frac{-(-6)}{2(2)} = \frac{6}{4} = \frac{3}{2}$

$k = \text{y-coord} = f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 7$

$= 2 \cdot \frac{9}{4} - 3 \cdot 3 + 7$

$= \frac{9}{2} - 9 + 7$

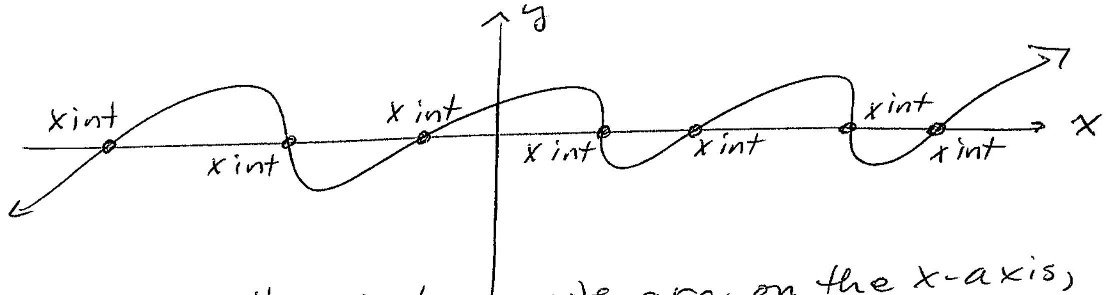
$= \frac{5}{2}$

$\boxed{\text{vertex } \left(\frac{3}{2}, \frac{5}{2}\right)}$ $\boxed{f(x) = 2\left(x - \frac{3}{2}\right)^2 + \frac{5}{2}}$ a = 2 unchanged

Math 60 10.5

x-intercepts:

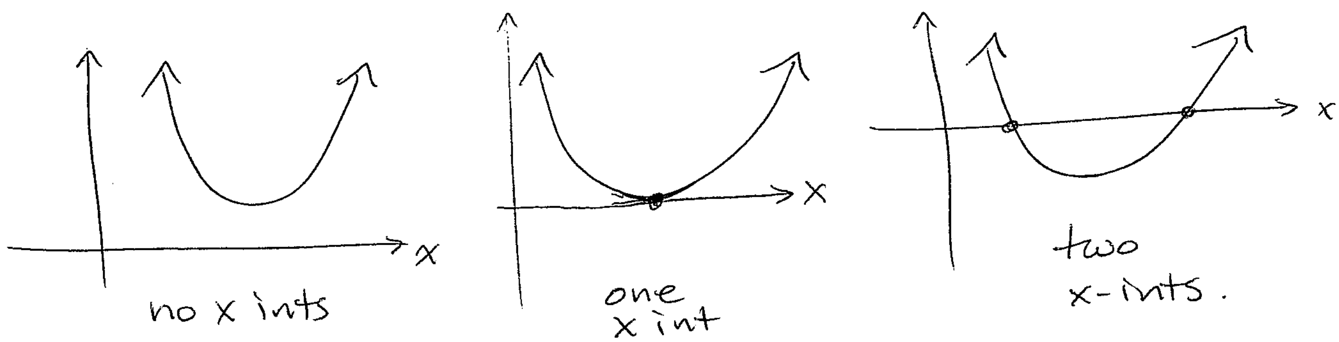
In general, x-intercepts are points where the graph crosses the x-axis.

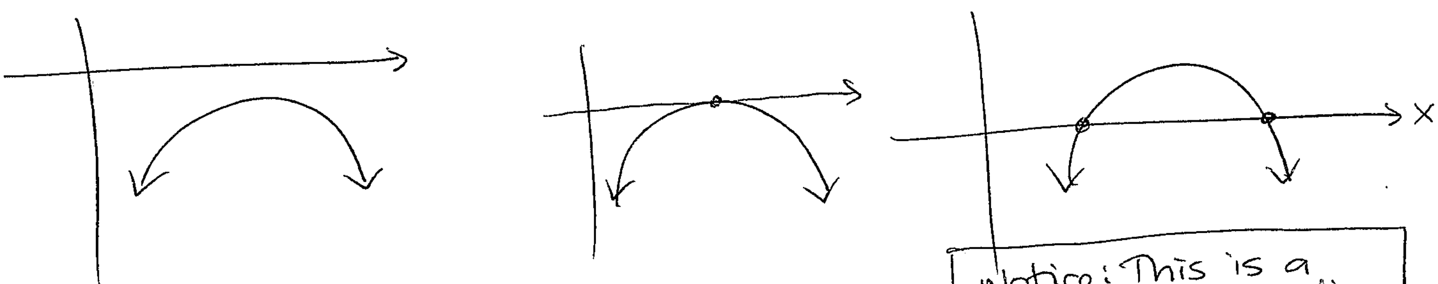


Because all x-intercepts are on the x-axis, their ordered pairs all have $y=0$.
To find x-intercepts, set $y=0$ (replace $f(x)$ by 0).

x-intercepts for quadratic functions

Possible scenarios: (graphically)





Notice: This is a quadratic equation not a quadratic function.

(algebraically): $0 = ax^2+bx+c$

has solutions $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

complex solutions no x-ints means $D=b^2-4ac<0$	one real solution one x-int $D=b^2-4ac=0$	two real solutions 2 x-ints $D=b^2-4ac>0$

Math 60 10.5

Use the discriminant to determine the number of x-intercepts, then find the x-intercepts.

(4) $f(x) = x^2 - 2x + 3$

$D = b^2 - 4ac$

$= (-2)^2 - 4(1)(3)$

$= 4 - 12$

$= -8$

negative = complex w/ imaginary $\Rightarrow$ $\boxed{\text{no x-ints}}$

(5) $g(x) = x^2 - 2x - 3$

$D = b^2 - 4ac$

$= (-2)^2 - 4(1)(-3)$

$= 4 + 12$

$= 16$

positive = 2 real solutions $\Rightarrow$ $\boxed{\text{2 x-ints}}$

$x = \dfrac{-b \pm \sqrt{D}}{2a}$

$x = \dfrac{-(-2) \pm \sqrt{16}}{2}$

$= \dfrac{2 \pm 4}{2}$

$= \dfrac{2+4}{2}, \dfrac{2-4}{2}$

$= \dfrac{6}{2}, \dfrac{-2}{2}$

$x = 3, -1$ $\boxed{\text{x ints } (3,0) \text{ and } (-1,0)}$

(6) $h(x) = 2x^2 - 6x + 7$

$D = b^2 - 4ac$

$= (-6)^2 - 4(2)(7)$

$= 36 - 56$

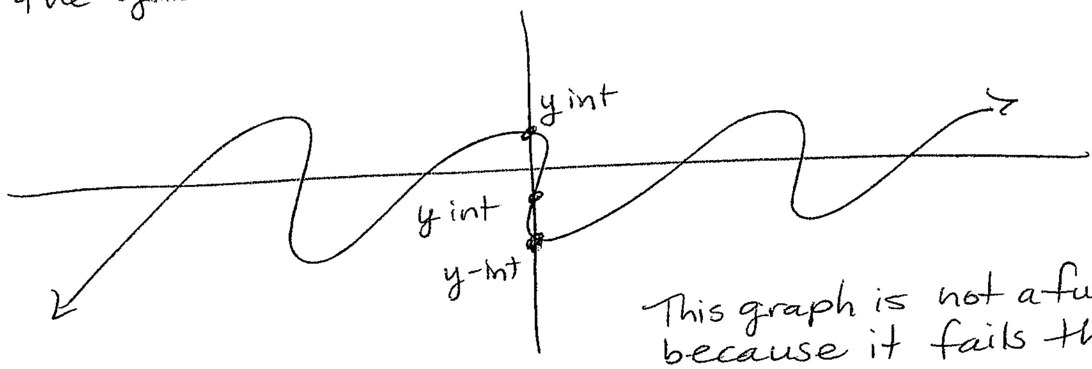
$= -20$

complex with imaginary $\Rightarrow$ $\boxed{\text{no x-ints}}$

Math 60 10.5

y-intercepts:

In general, y-intercepts are where the graph crosses the ~~y~~ y-axis.



This graph is not a function because it fails the V.L.T.

Because all y-intercepts are on the y-axis their ordered pairs all have $x=0$.

To find y-intercepts, set $x=0$.

y-intercepts for quadratic functions

All quadratic functions have exactly one y-intercept.

If $f(x) = ax^2 + bx + c$

Then $f(0) = a(0)^2 + b(0) + c$

$= c$

So y int $= (0, c)$.

Find y-intercepts.

7. $f(x) = x^2 - 2x + 3$

$f(0) = 3$

$\boxed{(0,3)}$

8. $g(x) = x^2 - 2x - 3$

$f(0) = -3$

$\boxed{(0,-3)}$

9. $h(x) = 2x^2 - 6x + 7$

$h(0) = 7$

$\boxed{(0,7)}$

Math 60 10.5

Graph each function

(10) $f(x) = x^2 - 2x + 3$

summary of our work in previous questions

vertex $(1,2)$

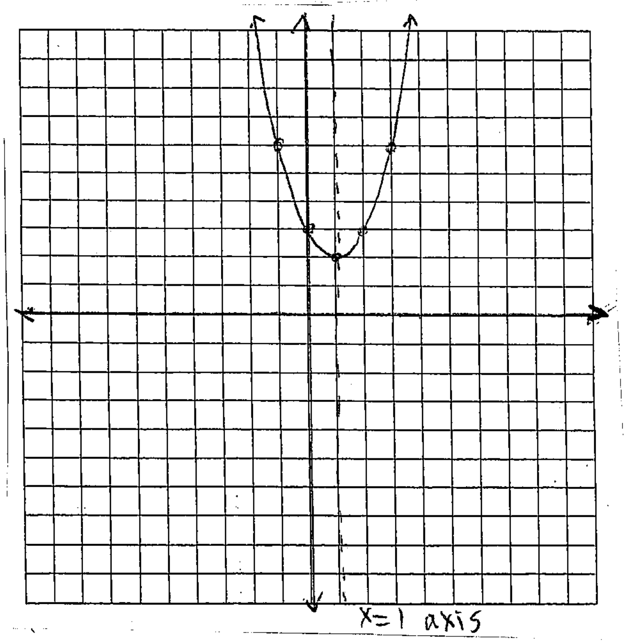
$f(x) = (x-1)^2 + 2$

no x-ints

y int $(0,3)$

opens up

$a=1$ standard shape



(11) $g(x) = x^2 - 2x - 3$

summary

vertex $(1,-4)$

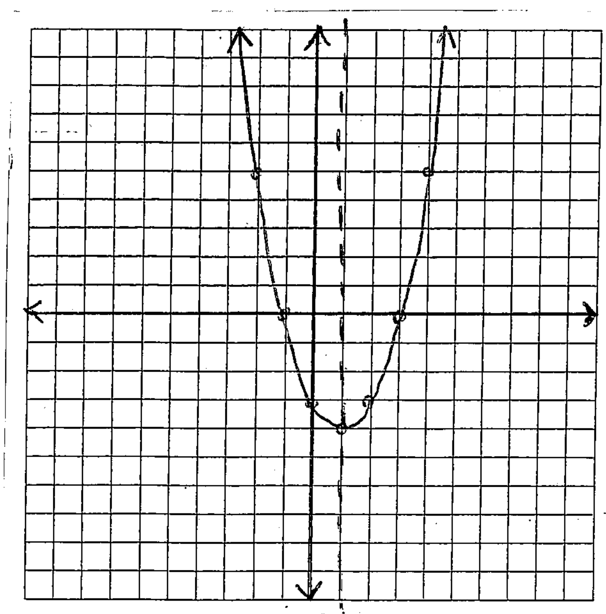
$f(x) = (x-1)^2 - 4$

2 x-ints $(3,0)$ and $(-1,0)$

y int $(0,-3)$

opens up

$a=1$ standard shape



Math 60 10.5

(12) $f(x) = 2x^2 - 6x + 7$

summary

vertex $\left(\frac{3}{2}, \frac{5}{2}\right)$

$f(x) = 2\left(x - \frac{3}{2}\right)^2 + \frac{5}{2}$

no x-ints

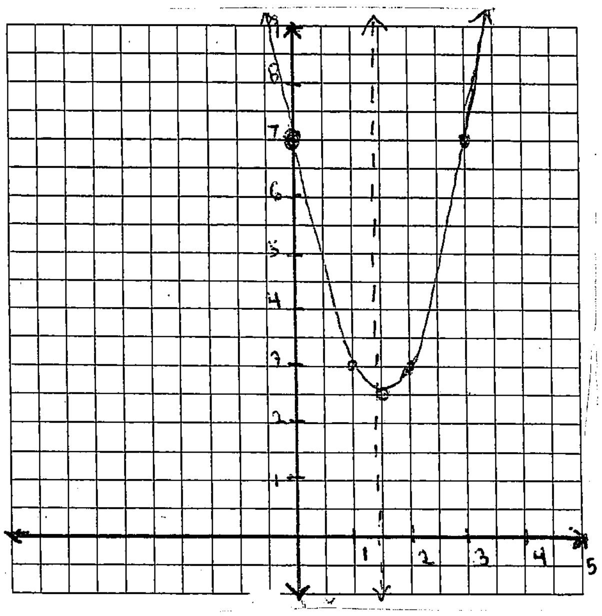
y int $(0,7)$

$a = 2$ narrower than standard

Because vertex has fractions, use scale 1 block $= \frac{1}{2}$ unit

y-int $(0,7) = \left(0, \frac{14}{2}\right) \leftarrow$ need 14 blocks above the x-axis.

notice that $(3,7)$ is the mirror image of $(0,7)$.



Useful point $x = 1$

$f(1) = 2(1)^2 - 6(1) + 7$

$= 2 - 6 + 7$

$= 3$

plot $(1,3)$

and its mirror image $(2,3)$

Math 60 10.5

When graphing in Ms. Carey's class:

- To graph a parabola, always plot at least 5 points including the vertex.
- Draw the axis of symmetry and use mirror images

  ↳ This means you really need only the vertex plus 2 other points on one side of the axis

- Draw axes – label scale if each box is something other than 1.
- Extend graph neatly and accurately to the edges of the grid.
- Parabolas should be round at the vertex (not pointy)

yes

No.

- work neatly!

skip.

(13) Graph $f(x) = -2x^2 - 8x + 1$ using its properties.

x int: $D = b^2 - 4ac$

$= (-8)^2 - 4(-2)(1)$

$= 64 + 8$

$= 72$ 2 irrational ⇒ crosses x-axis twice but at icky #s.

Vertex: $h = \dfrac{-b}{2a}$

$= \dfrac{-(-8)}{2(-2)}$

$= \dfrac{8}{-4}$

$= -2$ x-coordinate of vertex is -2

$k = f(h) = f(-2)$

$= -2(-2)^2 - 8(-2) + 1$

$= -8 + 16 + 1$

$= 9$ y-coord of vertex is 9

vertex $(-2, 9)$

Math 60 10.5

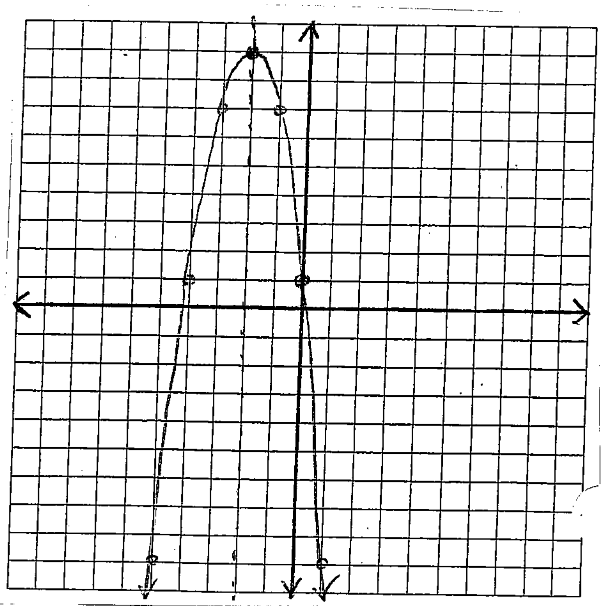
<u>axis of symmetry:</u> $x = -2$

<u>$a = -2$:</u> opens down $a < 0$.

$|a| = 2 > 1$ narrower than standard.

<u>y-intercept:</u> $f(0) = -2(0)^2 - 8(0) + 1$

$(0, 1)$.



Plot vertex.

Draw axis of symmetry

Plot y-int

Plot mirror image of y-int at $(-4, 1)$.

Useful point $x = -1$

$f(-1) = -2(-1)^2 - 8(-1) + 1$

$= -2 + 8 + 1$

$= 7$

plot $(-1, 7)$

Plot mirror image of $(-1, 7)$ at $(-3, 7)$

Useful point $x = 1$

$f(1) = -2(1)^2 - 8(1) + 1$

$= -2 - 8 + 1$

$= -9$

Plot $(1, -9)$

Plot mirror image of $(1, -9)$ at $(-5, -9)$

Connect smoothly, round at vertex.